

DISCRETE MATHEMATICS SEMINAR
CENTER FOR APPLIED MATHEMATICS AND SCIENCE
DEPARTMENT OF MATHEMATICS
UNIVERSITY OF WEST GEORGIA

2:00 - 2:50 PM, FRIDAY, NOVEMBER 13, 2015

BOYD 306

Speaker: **Dr. Dong Ye**

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Title: **Connectivity and non-revisiting paths in polyhedral maps on surfaces**

Abstract:

The W_v -path conjecture states that any two vertices of a simple polytope can be joined by a path that does not revisit any facet. This is equivalent to the well-known Hirsch Conjecture. Klee conjectured even more, namely that the W_v -conjecture is true for all general cell complexes. Klee proved that the W_v -conjecture is true for 3-polytope (3-connected plane graphs). Later, the general W_v -conjecture was verified for polyhedral maps on the projective plane and torus by Barnette, and on the Klein-bottle by Pulapaka and Vince. Recently, however, Santos proved that the Hirsch conjecture is false.

In this talk, we will present some recent development on the W_v -path problem in polyhedral maps on surfaces. We show that the W_v -path problem is closely related to both the local connectivity $\kappa_G(x, y)$, and the number of non-homotopy classes of (x, y) -paths as well as the number of (x, y) -paths in each homotopy class. For every surface Σ , define a function $f(\Sigma)$ such that if for every graph polyhedrally embedded in Σ and for every pair of vertices x and y in $V(G)$, $\kappa_G(x, y) \geq f(\Sigma)$, then there exists a W_v -path joining x and y . We show that $\max\{3, 1 - \chi(\Sigma)\} \leq f(\Sigma) \leq 9 - 4\chi(\Sigma)$, where $\chi(\Sigma)$ is the Euler characteristic. Further, if x and y are not co-facial, we show that G has at least $\kappa_G(x, y) + 4\chi(\Sigma) - 8$ internally disjoint W_v -paths joining x and y . The bound is sharp for the sphere.

This is joint work with Michael D. Plummer and Xiaoya Zha.